Analysis of a Simple Recruiting Method for Cooperative Routes and Strip Networks

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Abstract

The Opportunistic Large Array (OLA) is the basis for a simple cooperative transmission protocol that is suitable for broadcasting in wireless sensor networks and in which every node transmits once per broadcast. OLA with Transmission Threshold (OLA-T) is an energy-efficient extension of the OLA protocol that limits node participation during broadcasting. Performance of OLA-T has been studied for disc-shaped networks. In this paper, we propose a method to systematically set the transmission threshold and design the OLAs for two-dimensional strip-shaped networks. Theoretical bounds and conditions for achieving sustained OLA propagation and reducing the total energy consumption in the network using OLA-T have been derived in this paper. These results would also apply to arbitrarily shaped networks that have node participation limited to strip-shaped collections.

I. INTRODUCTION

Cooperative Transmission (CT)-based strategies enable collections of single-antenna radios to achieve the spatial diversity benefits of an array transmitter, and can be used to save energy and achieve range-extension [1], [2]. The proposed method is based on a simple form of cooperative transmission called the Opportunistic Large Array (OLA) [3], which applies to high-density wireless sensor networks (WSNs). While price is still a barrier to high-density WSNs, continued development in ultra-low power and energy harvesting radios could someday enable large, dense multi-hop networks [4]. Such networks might be deployed on structures that are strip-shaped,

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for example, dense distributions of wireless strain gages may be deployed on bridges in the structural health monitoring application [5]. Alternatively, a strip may occur as a "cooperative route," within a larger, dense multi-hop network. A cooperative route is a set of nodes that are candidates for cooperation between a source and a destination; the set is many hops long and multiple nodes wide, and may be constructed based on a conventional multi-hop route [6], or by using other means, such as the OLACRA [7] and OLA-ROAD [8] protocols, which do not require an existing conventional route. This paper assumes that the strip-shaped candidate set already exists, and provides a simple and systematic way for the cooperators along such routes to be selected. "Basic OLA" for the strip network was studied in [9]. This paper extends that work to include a *user-defined* transmission threshold, which is a mechanism to limit node participation and save energy.

An OLA is a group of single-antenna relays or forwarding nodes that operate without any mutual coordination, but naturally fire transmit diversity waveforms in independently fading orthogonal channels approximately simultaneously in response to energy received from a single source or another OLA [3]. OLA transmission time synchronization with the root mean square transmit time delay spreads less than 100 ns have been demonstrated [10]. One simple, power amplifier-friendly way to achieve transmit diversity is to transmit On-Off-Shift Keying (OOK) or Frequency-Shift Keying (FSK) on orthogonal carriers, with a simple energy detectors in the receiver [10]. Even though many nodes may participate in an OLA transmission with diversity, total transmission energy can still be saved because all nodes can reduce their transmit powers dramatically and large fade margins are not needed.

When using "Basic OLA" for broadcasting in a network, the nodes repeat if they haven't repeated the packet before, and the OLAs will "propagate," eventually including all the nodes in the network, under a condition on relay power and receiver sensitivity [3]. In [3], Basic OLA was shown to yield an energy savings of about 5 dB compared to a popular broadcast tree algorithm, the Broadcast Incremental Power (BIP) algorithm [11]. OLA-T is the same as Basic OLA except each node applies a Signal-to-Noise Ratio (SNR) threshold to limit the relaying nodes to those at the edge of the decoding range [12], [13]. Even though these "border nodes" must transmit at a higher power than for Basic OLA to sustain propagation [13], total transmit energy is still saved because in Basic OLA, nodes that are not at the border must waste some transmission energy just getting their signals over other nodes that have already received the signal.

OLA-T is distinct from cooperative medium access control (MAC) protocols that use SNR as a basis for relay participation [14]–[17]. In most proposed CT MAC protocols, potential relays must (i) learn the quality of their links to the destination via feedback [14]–[16], and (ii) contend for the channel, using, for example, a priority-based contention window size to favor the preferred relays [14]. In contrast, OLA-T is a purely feed-forward approach (there is no pre-determined destination for each intermediate OLA-T hop) and there is no contention among relays, because in any particular hop, the relays transmit synchronously. The paper by [17] uses an SNR threshold on only the source-relay signal. However, [17] assumes no decoding error detection at the relay (e.g. CRC check) and requires that a relays SNR must be higher than the threshold, while the present paper does the opposite on both points; furthermore [17] analyzes the error probability of a single link whereas this paper analyzes the requirements for successful transmission over an unlimited number of hops.

II. System Model

For our analysis, we adopt the notation and assumptions of [13], some of which were used earlier in [9]. Half-duplex nodes are assumed to be distributed randomly and uniformly over a continuous strip defined by $\mathbb{S} = \{(x, y) : |y| \leq \frac{W}{2},\ 0 \leq x \leq L\}$ with average node density ρ, width W, and length L.

We assume a node can *decode and forward* (DF) a packet without error when its received signal-to-noise ratio (SNR) is greater than or equal to a modulation-dependent 'lower' threshold, τ_l [13]. In practice, there is no explicit SNR threshold; nodes DF only if they pass CRC check. In contrast, the 'Transmission' or 'upper' threshold, τ_u is used explicitly in the receiver to compare against the received SNR. This additional criterion for relaying limits the number of nodes in each hop because a node would relay only if its received SNR is *less* than τ_u. Assumption of unit noise variance transform τ_l and τ_u to received powers, which define a range of received powers that correspond to the "significant" boundary nodes, which form the OLA. We define the Relative Transmission Threshold (RTT) as $\mathcal{R} = \frac{\tau_u}{\tau_l}$.

For simplicity, the *deterministic model* [9] is assumed, which means that the power received at a node is the sum of the powers from each of the node transmissions. This implies that signals received from different nodes are orthogonal. Techniques to *induce* orthogonality in the node transmissions by randomly delaying the firing times (such as in [18]) or transmitting on orthogonal carriers (Frequency Division Multiplexing) will work as long as the receivers can extract the multipath diversity from the wireless channel.

Following the Continuum Model for a strip network from [9], we assume a non-fading environment and a path-loss exponent of 2. The path-loss function in Cartesian coordinates is given by $l(x, y) = (x^2 + y^2)^{-1}$, where (x, y) are the normalized coordinates at the receiver. As in [13], distance d is normalized by a reference distance, d_0. Let the normalized source and relay transmit powers be denoted by P_s and P_r, respectively, and the relay transmit power per unit area be denoted by $\overline{P_r} = \rho P_r$. The normalization is such that P_s and P_r are actually the SNRs at a receiver d_0 away from the transmitter [13]. Since we assume a continuum of nodes in the network, we let the node density ρ become very large ($\rho \to \infty$) while $\overline{P_r}$ is kept fixed. Our results are parameterized by $\mathcal{R}$ and node degree, $\mathcal{K}$, which for any finite node density, can be expressed as $\mathcal{K} = \pi\overline{P_r}/\tau_l$ [7]. Columns 2–6 of Table I give some example values of our key parameters.

III. OLA-T for the Strip Network

In this section, the analytical framework for deriving the necessary and sufficient conditions for sustained broadcast using OLA-T is developed. Fig. 1 represents the propagation of a packet along a strip network using OLA with Transmission Threshold (OLA-T). The source node, S, initiates the packet transmission and all the nodes in the vicinity of the source node that can decode the packet form the *first* Decoding Level, $\mathbb{D}_1$. The nodes in $\mathbb{D}_1$ that satisfy the transmission threshold constitute the "OLA-1" nodes or the first OLA, and are denoted by $\mathbb{S}_1$ in Fig. 1. Mathematically, $\mathbb{S}_1 = \{(x, y) \in \mathbb{S} : \tau_l \leq \int\int P_s\ l(x, y)dxdy \leq \tau_u\}$.

Next, the set of nodes in the vicinity of OLA 1 that decode the packet, but have not previously decoded the same packet, form the *second* Decoding Level, $\mathbb{D}_2$. Again, *only* the nodes in $\mathbb{D}_2$ that satisfy the transmission threshold constitute the "OLA-2" nodes, denoted by $\mathbb{S}_2$ in Fig. 1. In general, $\mathbb{S}_k = \{(x, y) \in \mathbb{S}\backslash \bigcup_{i=1}^{k-1} \mathbb{D}_i : \tau_l \leq \overline{P_r} \displaystyle\int\int_{\mathbb{S}_{k-1}} l(x - x', y - y')dx'dy' \leq \tau_u\}$ is used to define the OLA-k nodes.

A. Rectangular Approximation

We assume that the width of the strip, W, the hop distance, $r_{o,k}$, and the OLA lengths, d_k, of the k-th hop, are such that the 'curved' decoding ranges (the regions between the solid and dash-dot lines) $\mathbb{S}_k$ can be approximated by the 'shaded' rectangles $\tilde{\mathbb{S}}_k$ shown in Fig. 1. In Table I, Columns 8–10 correspond to hop index $>> 1$ or steady state. It can be seen that the straight-line approximation results in low approximation errors in Examples 1, 2, 3, and 5, while Examples 4 and 5 show that a smaller W yields a better approximation. We also observe that the high density cases, Examples 1, 2, and 3, correspond to very low transmit powers.

With this approximation, we can derive the boundaries for the k-th OLA for the OLA-T protocol. The inner and outer boundaries that define the OLA-1 nodes are $r_{i,1}$ and $r_{o,1}$, respectively, as shown in Fig. 1. Using the definition of the path-loss functions defined previously, $r_{i,1} = \sqrt{\frac{P_s}{\tau_u}}$ and $r_{o,1} = \sqrt{\frac{P_s}{\tau_l}}$. $\tilde{\mathbb{S}}_1$ is the first OLA with boundary conditions given by $\sqrt{\frac{P_s}{\tau_u}} \leq x \leq \sqrt{\frac{P_s}{\tau_l}}$ and $|y| \leq \frac{W}{2}$. We denote the length of the first OLA as d_1, given by $d_1 = r_{o,1} - r_{i,1} = \sqrt{\frac{P_s}{\tau_l}} - \sqrt{\frac{P_s}{\tau_u}}$.

In order to approximate the curved inner and outer boundaries for $\mathbb{S}_2$ by straight lines, $r_{i,2}$ and $r_{o,2}$ are chosen to satisfy $\overline{P_r} \int\int_{\tilde{\mathbb{S}}_1} l(x - [r_{i,1} + d_1 + r_{\Omega,2}], y) dx dy = \tau_\Gamma$, where $\Gamma = u$ when $\Omega = i$ and $\Gamma = l$ when $\Omega = o$. We observe from the previous equation, and Fig. 1, that $r_{i,2}$ and $r_{o,2}$ are both defined relative to $r_{o,1}$. By substituting the definition for $l(x, y)$ and making the limits explicit, we can write, $\int_{-W/2}^{W/2} \int_{r_{o,2}}^{r_{o,2}+d_1} \frac{\overline{P_r}}{x^2 + y^2} dx dy = \int_{r_{o,2}}^{r_{o,2}+d_1} \frac{2\overline{P_r}}{x} \arctan\left(\frac{W}{2x}\right) dx = \tau_l$. Similarly, $\int_{r_{i,2}}^{r_{i,2}+d_1} \frac{2\overline{P_r}}{x} \arctan\left(\frac{W}{2x}\right) dx = \tau_u$. So, $\tilde{\mathbb{S}}_2$ is the second OLA with a length $d_2 = r_{o,2} - r_{i,2}$. In this way, the subsequent OLA lengths $d_3, d_4, \ldots$ can be found iteratively $d_k = r_{o,k} - r_{i,k} = h_o(d_{k-1}) - h_i(d_{k-1})$, where the functions $h_\Omega(d_{k-1})$ for $d_{k-1} > 0,\ \ \Omega \in \{i, o\}$ are defined as the unique solutions of $\int_{h_\Omega(d_{k-1})}^{h_\Omega(d_{k-1})+d_{k-1}} \frac{2\overline{P_r}}{u} \arctan\left(\frac{W}{2u}\right) du = \tau_\Gamma$, where $\Gamma = u$ when $\Omega = i$ and $\Gamma = l$ when $\Omega = o$. We denote $h_o(\cdot) - h_i(\cdot) = g(\cdot)$. So, $d_{k+1} = g(d_k)$. The following properties for $g(\cdot)$ have been proved in the Appendix.

1) $\lim_{d \to 0} g(d) = 0$.
2) The function g is monotonically increasing.
3) The function g is concave downward.
4) The tangent at zero, $g'(0)$, is given by $g'(0) = h_o'(0) - h_i'(0) = \frac{1}{\exp\left(\frac{1}{\mathcal{K}}\right)-1} - \frac{1}{\exp\left(\frac{\mathcal{R}}{\mathcal{K}}\right)-1}$.
5) When $g'(0) > 1$, then g has a unique positive fixed point $g(d) = d$. When $g'(0) < 1$, the

only fixed point of g is at $d = 0$.

B. Sufficient and Necessary Conditions for Infinite OLA Propagation

Infinite propagation of the packet is determined by how the sum $\sum_k d_k$ grows with k. When this sum *is unbounded*, the OLAs (and hence, the packet) will propagate forever keeping the link between the source and destination intact irrespective of the distance between these points. However, if the sum is *finite*, then the packet does not reach the destination when the source and destination are too far apart.

The propagation of the packet along the strip network can be predicted by computing the slope of the *concave* function g at zero [9]. In particular, Properties (4) and (5) of $g(\cdot)$ imply analytical expressions for the two extreme cases: Transmissions reach a steady state when $g'(0) > 1$ and die out when $g'(0) < 1$. Equivalently, transmissions reach a steady state when $2 > \exp\left(\frac{1}{\mathcal{K}}\right) + \exp\left(\frac{-\mathcal{R}}{\mathcal{K}}\right)$. We observe that when $\mathcal{R} \to \infty$, $\exp\left(-\mathcal{R}/\mathcal{K}\right) \to 0$, OLA-T becomes Basic OLA, and the above equations become the conditions in [9]. Finally, the condition for sustained propagation can be re-written in terms of a lower bound for $\mathcal{R}$: $\mathcal{R}_{\text{lower bound}} = -\mathcal{K} \ln\left[2 - \exp\left(\frac{1}{\mathcal{K}}\right)\right]$. So, $\mathcal{R} < \mathcal{R}_{\text{lower bound}}$ results in very thin OLAs (fewer nodes) that are too weak to sustain infinite propagation and eventually die out. We observe that this is the same lower bound as for the infinite disc network in [13]. We note that a similar condition held for both the disc and strip networks using Basic OLA [3], [9].

IV. Numerical Results and Discussion

Fig. 2 shows the lower bound on RTT, $\mathcal{R}_{\text{lower bound}}$, in dB, versus the node degree, $\mathcal{K}$. It is observed that as $\mathcal{K}$ increases, the minimum size of the 'SNR window' decreases. For example, for $\mathcal{K} = 1$, the minimum transmission threshold is about 1.8 dB higher than the decoding threshold. While for $\mathcal{K} = 10$, the minimum transmission threshold is *only* 0.2 dB higher than the decoding threshold. It can also be inferred that theoretically, it is possible for OLA-T to achieve infinite network broadcast with an infinitesimally small $\mathcal{R}_{\text{lower bound}}$ and very high $\mathcal{K}$. However, a very small $\mathcal{R}_{\text{lower bound}}$ may not be very effective if there is too much uncertainty in the SNR or if the node density is not high enough.

Fig. 3 shows the two extreme cases of $g'(x)$, depending on the value of the slope at $x = 0$. To generate these results, a node degree, $\mathcal{K} = \pi$ was assumed, which resulted in $\mathcal{R}_{\text{lower bound}} = 1.476$ or 1.68 dB. Violation of the lower bound should correspond to $g'(0) < 1$. To check this, $\mathcal{R}$ was chosen to be 1.3 (1.13 dB) and 3 (4.77 dB), for the cases, $g'(0) < 1$ and $g'(0) > 1$, respectively. $g'(0) < 1 \Rightarrow \mathcal{R} < \mathcal{R}_{\text{lower bound}}$ case is denoted by the dotted curve in Fig. 3. The other extreme is when $g'(0) > 1 \Rightarrow \mathcal{R} > \mathcal{R}_{\text{lower bound}}$, and this is represented by the dash-circle curve in Fig. 3. A fixed-point attractor away from zero at about $x = 2$ can be observed for the dash-circle curve, ensuring that the transmissions don't die out.

A. Energy Evaluation of OLA-T

We use the fraction of *radiated* energy saved (FES) as the metric for comparing the energy-efficiency of OLA-T relative to Basic OLA. Under the continuum assumption, the total energy consumption is simply the area of the rectangular OLAs. The FES for the strip network is computed as follows. The radiated energy consumed by OLA-T in the first N levels is mathematically expressed as $E_{\text{rad(OT)}} = \overline{P_r}_{(\text{OT})} T_s W \sum_{k=1}^{N} d_k$, where $\overline{P_r}_{(\text{OT})}$ is the lowest value of $\overline{P_r}$ that would guarantee successful broadcast using OLA-T and T_s is the length of the packet in time units. The energy consumed by Basic OLA is $E_{\text{rad(O)}} = \overline{P_r}_{(\text{O})} T_s W r_{\text{strip}}$, where $r_{\text{strip}} = \sum_{k=1}^{N} r_{o,k}$, and $\overline{P_r}_{(\text{O})}$ is the lowest value of $\overline{P_r}$ that would guarantee successful broadcast using Basic OLA. So, FES can be expressed as: $\text{FES} = 1 - (E_{\text{rad(OT)}}/E_{\text{rad(O)}}) = 1 - (\mathcal{K}_{(\text{OT})} \ln 2 \sum_{k=1}^{N} d_k / r_{\text{strip}})$, where $\mathcal{K}_{(\text{OT})}$ is the node degree for OLA-T to guarantee successful broadcast when operating in its minimum power configuration.

In wireless sensor networks, the radiated energy does not always dominate the total energy budget. Let the total circuit-consumed energy (E_{cir}) consumed by the network be proportional to $E_{\text{rad(O)}}$: $\text{E}_{\text{cir}} = \alpha E_{\text{rad(O)}}$. Then, we can define the whole-energy fraction of energy saved (WFES) as follows: $\text{WFES} = 1 - \left(\frac{E_{\text{rad(OT)}} + \text{E}_{\text{cir}}}{E_{\text{rad(O)}} + \text{E}_{\text{cir}}}\right) = \left(\frac{1}{1+\alpha}\right)\left(1 - \frac{E_{\text{rad(OT)}}}{E_{\text{rad(O)}}}\right) = \frac{\text{FES}}{1+\alpha}$. For example, if the circuit-consumed energy in a relaying node is the same as its radiated energy, then $\alpha = 1$. When $\alpha = 0$, we only consider the radiated energy (i.e., WFES = FES), and when $\alpha \neq 0$, the circuit energy is some fraction of the radiated energy. Even though no hardware-defined radios exist today that can support OLA transmissions, we can still consider some existing radio profiles.

For example, for the XE1205, we have $\alpha \approx 0.22$, and for the nRF905, we have $\alpha \approx 0.4$ [20].

Fig. 4 shows WFES versus node degree, $\mathcal{K}_{(\mathrm{OT})}$ (on a logarithmic scale), for a strip network for different values of α and $N = 30$. For example, for $\alpha = 0$, at $\mathcal{K}_{(\mathrm{OT})} = 10$, FES is about 0.55. This means that at their respective lowest energy levels at $\mathcal{K}_{(\mathrm{OT})} = 10$, OLA-T saves about 55% of the radiated energy used by Basic OLA. On the other hand, when both the circuit and transmit energies are equal or $\alpha = 1$, and $\mathcal{K}_{(\mathrm{OT})} = 10$, the WFES is about 0.28, meaning that OLA-T saves about 28% of the total energy consumed during broadcast relative to Basic OLA, both protocols operating in their minimum power configurations. It is noted that WFES increases with $\mathcal{K}_{(\mathrm{OT})}$ and attains a maximum of about 62%. This is because high values of $\mathcal{K}_{(\mathrm{OT})}$ imply fewer nodes in every hop (i.e. very slender OLA strips), which reduces the overall energy consumption in the network during broadcast.

B. OLA-T for a Disc Compared to the Strip

Interestingly, the WFES in a strip network is *almost twice* that of the WFES for a disc network from [13]. Intuitively, the reason is that the OLA part of the Decoding Level 1 is a significantly larger portion of the whole for the disc compared to the strip. The analytical reasoning for this is presented below. Consider the WFES for just the *first* OLA for both networks, because the first level dominates in the comparison. Let $r_{i,1}$ and $r_{o,1}$ be the inner and outer boundaries for the first OLA, respectively, and let $d_1 = r_{o,1} - r_{i,1}$. We note that the values of these parameters are equal for disc and strip networks (see Section III.A). $\mathrm{WFES}_{\mathrm{strip}} = \left(1 - \frac{d_1}{r_{o,1}}\right)\left(\frac{1}{1+\alpha}\right)$, and $\mathrm{WFES}_{\mathrm{disc}} = \left(1 - \frac{r_{o,1}^2 - r_{i,1}^2}{r_{o,1}^2}\right)\left(\frac{1}{1+\alpha}\right)$. Observe that $\mathrm{WFES}_{\mathrm{disc}} = \left(1 - \frac{d_1}{r_{o,1}} \underbrace{\frac{(r_{o,1} + r_{i,1})}{r_{o,1}}}_{>1}\right)\left(\frac{1}{1+\alpha}\right) < \mathrm{WFES}_{\mathrm{strip}}$.

V. Conclusions

In this paper, we analyzed a simple form of a cooperative transmission protocol, the Opportunistic Large Array with Transmission Threshold (OLA-T), for broadcasting along a strip network. It was shown that as long as the transmission threshold is above a critical value, the packet is delivered to the destination regardless of the distance between the source and the destination. The OLA-T protocol limits the node participation in a strip and was found to save as much as 62% of the transmitted energy relative to Basic OLA, when both protocols operated in their lowest power configurations. Future work includes a detailed analysis of the protocol for different path-loss exponents and finite node density.

APPENDIX

The properties listed in Section III–a are proved below, using the same list indices.

1) In [9], it was shown that $\lim_{x \to 0} h_\Omega(x) = 0$, $\Omega \in \{i, o\}$. Since, $g(x) = h_o(x) - h_i(x)$, it follows that $\lim_{x \to 0}(h_o(x) - h_i(x)) = \lim_{x \to 0} g(x) = 0$.
2) In order to show $g'(\cdot) > 0$, we differentiate with respect to x, to get $g'(x) = h_o'(x) - h_i'(x)$. From [9], we know that $h_\Omega'(x) = \frac{U(h_\Omega(x)+x)}{U(h_\Omega(x)) - U(h_\Omega(x)+x)}$, where $\Omega = \{i, o\}$ and $U(x) = \frac{1}{x}\text{arctan}\left(\frac{1}{2x}\right)$. Since $U(\cdot)$ is a decreasing function for $x > 0$, we also know that $h_o(x)$ and $h_i(x)$ are increasing functions in x. We use h_o and h_i instead of $h_o(x)$ and $h_i(x)$, respectively, for the sake of brevity. Further simplification results in the following closed-form expression for $g'(x)$: $g'(x) = \frac{U(h_i)U(h_o+x) - U(h_o)U(h_i+x)}{[U(h_o)-U(h_o+x)]\cdot[U(h_i)-U(h_i+x)]}$. The denominator is a product of *positive* terms, and so in order to complete the proof, it suffices to show that $U(h_i)U(h_o + x) - U(h_o)U(h_i + x) > 0 \Rightarrow \frac{U(h_i)}{U(h_i+x)} > \frac{U(h_o)}{U(h_o+x)}$. In other words, we need to show that $q(h, x) := \frac{U(h)}{U(h+x)}$ is decreasing in 'h'. However, because h_i and h_o are difficult to obtain, we computed them using iterative numerical methods. Thus, $U(h_i)$, $U(h_o)$, $\frac{U(h_i)}{U(h_i+x)}$, and $\frac{U(h_o)}{U(h_o+x)}$ are also computed numerically, upon which it can be verified that $q(h, x)$ is decreasing in h $\forall x > 0$. So, the inequality holds proving that g is a monotonically increasing function.
3) Numerically, it is found that $h_o(x)$ and $h_i(x)$ are concave downward functions, ie., $h_o''(x) < 0$ and $h_i''(x) < 0$. From [9], $h_\Omega''(x) = \underbrace{\frac{U^2(h_\Omega + x)U^2(h_\Omega)}{[U(h_\Omega) - U(h_\Omega + x)]^3}}_{=:v(h_\Omega,x)} \cdot \underbrace{\left[\frac{U'(h_\Omega + x)}{U^2(h_\Omega + x)} - \frac{U'(h_\Omega)}{U^2(h_\Omega)}\right]}_{=:w(h_\Omega,x)}$, where $\Gamma = u$ when $\Omega = i$ and $\Gamma = l$ when $\Omega = o$. Since closed-form expressions for h_i and h_o are very difficult to obtain, $v(h, x)$ and $w(h, x)$ were computed numerically. It can be verified that the product $v(h, x) \cdot w(h, x)$ is decreasing in h, i.e., $v(h_i, x) \cdot w(h_i, x) > v(h_o, x) \cdot w(h_o, x)$ $\forall x > 0$. Thus, $g''(\cdot) < 0$, implying that g is concave downward.
4) Using the results from [9], $h_o'(0) = \frac{1}{\exp\left(\frac{1}{\mathcal{K}}\right)-1}$ and $h_i'(0) = \frac{1}{\exp\left(\frac{\mathcal{R}}{\mathcal{K}}\right)-1}$. Since $g'(0) = h_o'(0) - h_i'(0)$, (5) follows.

5) We know that $g'(x) = h'_o(x) - h'_i(x)$. Since $h'_\Omega(x) \xrightarrow{x\to\infty} 0,\ \Omega \in \{o,i\}$, it follows that $g'(x) \to 0$ as $x \to \infty$. If $g'(0) > 1$, then $g(x) > x$ for all $x > 0$ small enough. Since, $g(\cdot)$ is increasing and $g'(x) \to 0$, $g(x) < x$, for x large enough, i.e. the local attractor is away from the origin. On the other hand, when $g'(0) < 1$, $g(x) < x$ for sufficiently small $x > 0$. From the concavity of g, it follows that $g(x) = x$ can happen only at $x = 0$, i.e. the local attractor is the origin. Finally, in [9], the existence and uniqueness of $h_\Omega(\cdot)$ for $\Omega \in \{o,i\}$ was proved. Using the property that $g(\cdot)$ is monotonically increasing, it follows that for $x_1 \neq x_2$, $g(x_1) \neq g(x_2)$, i.e., the solutions to $g(\cdot)$ are unique, and when $x_1 \neq x_2$, $g(x_1) = g(x_2)$, it just implies that the OLA propagation continues with fixed step sizes after the initial transient phase.

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TABLE I

EXAMPLES OF UN-NORMALIZED VARIABLES

Example	P_r (dBm)	Node Density (nodes/area)	RX sens. (dBm)	$\mathcal{K}$	$\mathcal{R}$ (dB)	W	$r_{o,\infty}$	d_∞	% error in asymptotic areas
1	-48.00	3 nodes/m^2	-90.00	12.56	1.2	10 m	49.6 m	2.89 m	1.12
2	-48.00	18 nodes/m^2	-94.77	2	2.5	2 m	14.50 m	2.45 m	2.15
3	-56.00	10 node/m^2	-90.00	7.85	2.26	5 m	38.70 m	2.77 m	1.58
4	-20.97	2.5 nodes/km^2	-90.00	7	1	10 km	54.92 km	4.32 km	6.76
5	-20.97	2.5 nodes/km^2	-90.00	7	1	3 km	54.92 km	4.32 km	2.04

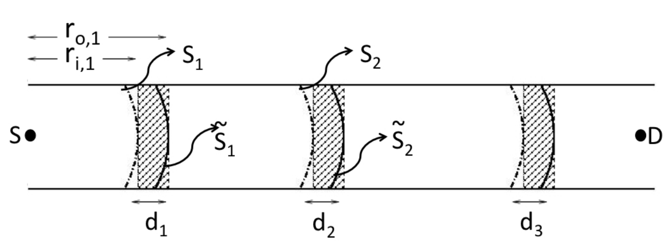

Fig. 1. Propagation along a network strip using OLA-T with a *straight line* approximation.

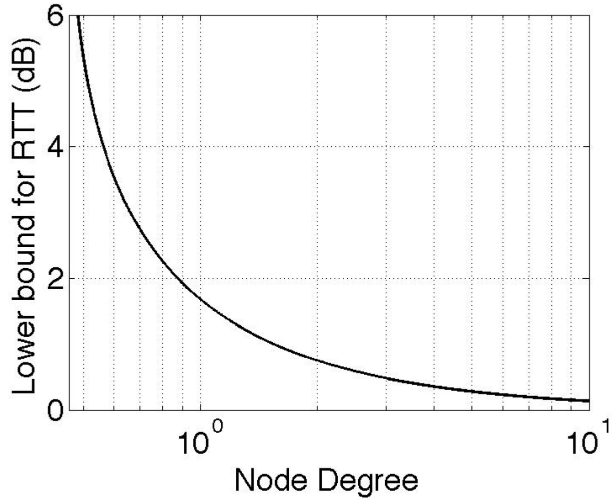

Fig. 2. Lower bound on RTT, $\mathcal{R}_{\text{lower bound}}$, in dB versus node degree, $\mathcal{K}$.

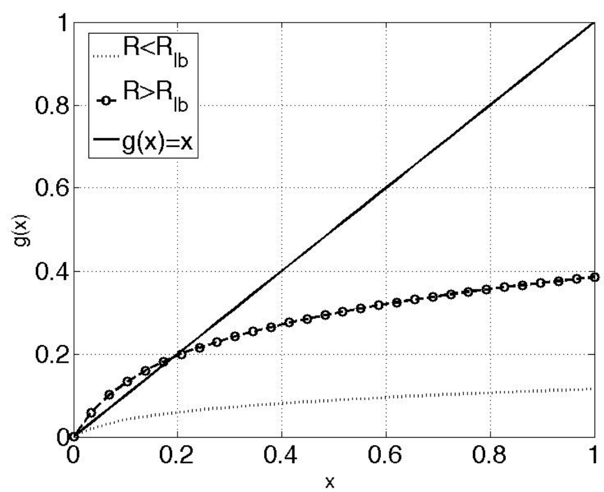

Fig. 3. $g(x)$ versus x for the three cases; $g'(0) < 1$ and $g'(0) > 1$.

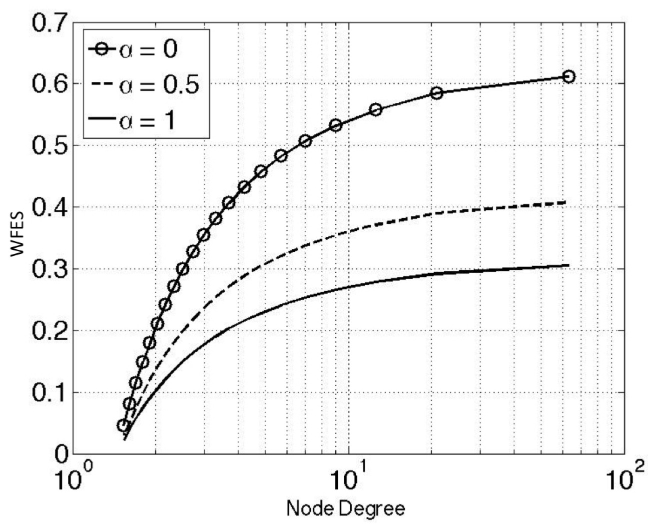

Fig. 4. Variation of FES with the node degree, $\mathcal{K}_{(\mathrm{OT})}$.