Establishing Performance Bounds for Alternating Cooperative Broadcasts Under High Path-Loss

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***Abstract*—This paper addresses the issue of smart network broadcasting using alternating concurrent cooperative transmissions for sensor-based wireless networks that are very lossy. This medium access control (MAC)-free broadcast strategy is a simple, energy-efficient, low-overhead form of cooperative diversity called the opportunistic large array (OLA), and uses a received power-based threshold to define mutually exclusive sets of nodes, such that the union of the sets includes all the nodes in the network or cooperative route during the initial broadcast. This protocol intelligently eliminates the formation of undesirable network coverage holes, a result of the "dead" node resulting from repeated usage. The semi-analytical approach in the paper investigates network life extensions for two extreme continuum network topologies that correspond to the largest and smallest ratios of nodes (or areas) used up during a successful broadcast, namely discs and strips, respectively for $\gamma > 2$. Analyzing the performance of this broadcast strategy for these two contrasting topologies will then set the bounds for arbitrary-shaped static routes or networks for $\gamma > 2$.**

I. Motivation

Body area networks (BANs) and short-range indoor links are examples of very lossy communication media, where the path loss exponents range between 3 and 7, a lot different from the free space propagation exponent (i.e., $\gamma = 2$) [1], [2]. The high path loss impacts the energy consumption, and subsequently, the network operation lives. At higher path losses, cooperative diversity-based approaches become advantageous and sometimes an absolute requirement to maintain the connectivity of the system. Cooperative diversity-based strategies leverage the spatial diversity in a network by having multiple nodes transmit the same message, and offer an signal-to-noise ratio (SNR) advantage in a multi-path fading environment that could be used to lower total transmission power expended in the network, achieve range extension, etc., [3], [4]. The energy consumption can be decreased in such harsh conditions because the transmission effort is spread over the whole network. In this paper, we focus on a simple, low-overhead form of cooperative diversity-based strategy called the opportunistic large arrays (OLAs), wherein a group of nodes learn their levels "on the fly," and autonomously fire at approximately the same time the transmit diversity waveforms in response to energy received from a single source or another OLA [5], and we will refer to this as "Basic OLA" in this paper. OLA with a transmission threshold (OLA-T), an energy-efficient version of Basic OLA, applies a received signal power-based threshold to recruit cooperators at the edge of the decoding range [6], [7]. The alternating OLA-T (A-OLA-T), a non-trivial extension of OLA-T, intelligently overcomes the formation of dead zones in a network by avoiding using the same sets of cooperating nodes during successive broadcasts [8].

Contributions: In this paper, we investigate the impact of higher path-attenuation on the broadcast performance of a smart, alternating, concurrent, cooperative transmissions-based protocol, namely the A-OLA-T protocol. When the path loss exponent increases, one might expect the border nodes to dominate the OLA transmission energy even more, thereby widening the gap in the energy consumption between Basic OLA and OLA-T, and hence, the network lives [1] between Basic OLA and A-OLA-T. This strongly motivates investigation of A-OLA-T in environments with higher (> 2) path loss

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[1] Defined in this paper as the time until the first node in the network dies.

exponents. Bounds on the network life extensions offered by A-OLA-T relative to Basic OLA as a function of key network parameters at higher path loss exponents have been derived semi-analytically, and are the original contributions of this paper. Two contrasting and extreme network topologies corresponding to the largest and smallest ratios of nodes (or areas) used up during a successful broadcast are considered for the analysis to set the performance bounds for arbitrary-shaped cooperative routes or networks.

Related Work: Dividing the nodes in the network into disjoint sets and activating them successively one set at a time to carry out network functions (e.g., monitoring, target tracking, etc.) has been investigated for non-cooperative wireless networks [9]–[11]. In [9]–[10], the network is divided into sets with the objective of maximizing the coverage (in terms of sensing area or targets-tracking) of each set, and the centralized algorithms activate these sets, one at a time, using only the sensors from the current active set for monitoring all targets and for transmitting the collected data to a sink. More recently, [12] explored combining and alternating between two non-cooperative routes (formed using multi-path routing and spanning tree algorithms) to prolong the operation life of nodes in a sensor network. Compared to the aforementioned non-cooperative strategies, in A-OLA-T, the sets are formed proactively based on the received signal power.

II. Analytical Framework

This section describes the topologies used to analyze A-OLA-T at higher path loss exponents, and to derive performance bounds and conditions for sustained operation. The disc- and strip-shaped cooperative routes (networks) correspond to the largest and smallest ratios of nodes (or areas) used up during a successful broadcast, respectively, and in this paper A-OLA-T is analyzed for these two scenarios. Analyzing this cooperative diversity-based protocols for these two contrasting and extreme network topologies will then set the performance bounds for arbitrary-shaped routes or networks. Half-duplex nodes are assumed. For the purpose of analysis, the nodes are assumed to be distributed uniformly and randomly over a continuous area with average density ρ. The originating node is assumed to be a point source at the center of the given network area. It is assumed that a node can decode and forward a message without error when its received SNR is greater than or equal to a modulation-dependent threshold [13]. The assumption of unit noise variance transforms the SNR threshold to a received power criterion, which is denoted as the decoding threshold τ_l. It should be noted that the decoding threshold τ_l is not explicitly used in real receiver operations. A real receiver always just tries to decode a message. If no errors are detected, then it is assumed that the receiver power must have exceeded τ_l.

In contrast, the OLA-T uses a "user-defined" transmission or 'upper' threshold, τ_u that is explicitly compared to an estimate of the received SNR. This additional criterion for relaying limits the number of nodes in each hop because a node would relay only if its received SNR is *less* than τ_u. So the thresholds, τ_l and τ_u, define a range of received powers that correspond to the "significant" boundary nodes, which form the OLA. We define the relative transmission threshold (RTT), $\mathcal{R}$, as the ratio between the two thresholds. Mathematically, $\mathcal{R} = \frac{\tau_u}{\tau_l}$. It is noted that $\mathcal{R} > 1$. While each boundary node in OLA-T must transmit a somewhat higher power, compared to Basic OLA, there is still an overall transmit energy savings with OLA-T because of the favorable location of the boundary nodes.

For simplicity, the *deterministic model* [13] is assumed, which means that the power received at a node is the sum of the powers from each of the node transmissions. This implies that signals received from different nodes are orthogonal. Techniques to *induce* orthogonality in the node transmissions by randomly delaying the firing times (such as in [14]) or transmitting on orthogonal carriers (frequency division multiplexing) will work as long as the receivers can extract the multi-path diversity from the wireless channel. Let the normalized source and relay transmit powers be denoted by P_s and P_r, respectively, and the relay transmit power per unit area be denoted by $\overline{P_r} = \rho P_r$. The normalization is such that P_s and P_r are actually the SNRs at a receiver d_0 away from the transmitter [6]. Since we assume a continuum of nodes in the network, we let the node density ρ become very large ($\rho \to \infty$) while $\overline{P_r}$ is kept fixed. Our results are parameterized by $\mathcal{R}$ and node degree, $\mathcal{K}$, which is the average number of nodes in the decoding range of a transmitter. Mathematically, for any finite node density, $\mathcal{K} = \pi \overline{P_r}/\tau_l$.

III. Performance Evaluation at Higher Path Loss Exponents

Under the deterministic path loss model, the concentric ring structure of the OLA propagation is still preserved. The outer and inner boundary radii for the k-th OLA ring are denoted as $r_{o,k}$ and $r_{i,k}$, respectively. Realizing that every broadcast in A-OLA-T is an OLA-T broadcast, the radiated energy consumed in a single broadcast in the first L levels for a continuum case is mathematically expressed, in energy units, as

$E_{\text{rad(OT)}} = \overline{P_r}_{(\text{OT,min})} T_s \sum_{k=1}^{L} \pi(r_{o,k}^2 - r_{i,k}^2)$, where T_s is the length of the message in time units and $\overline{P_r}_{(\text{OT,min})}$ is the lowest value of $\overline{P_r}$ that would guarantee successful broadcast using OLA-T. The energy consumed by Basic OLA is given by $E_{\text{rad(O)}} = \overline{P_r}_{(\text{O,min})} T_s \pi r_{o,L}^2$, where $\overline{P_r}_{(\text{O,min})}$ is the lowest value of $\overline{P_r}$ that would guarantee successful broadcast using Basic OLA. Because of the continuum assumption, the fraction of network life extension (FLE) for A-OLA-T relative to Basic OLA can be expressed as $\text{FLE} = \frac{E_{\text{rad(O)}}}{E_{\text{rad(OT)}}}$, and in terms of relative areas as $\big\{(\text{ratio of areas}) \times (\text{ratio of minimum node degrees})\big\}^{-1}$.

Limitations of Analytical Approach: The minimum node degrees for Basic OLA and OLA-T given by [6] and [13], respectively, hold only for $\gamma = 2$, and need to be evaluated for higher path loss exponents. Also, the radii definitions for computing the ratio of areas also depend on γ. Thus, both the ratios in the expression for FLE (from above) depend on γ, implying that FLE depends on γ. The parameters of interest can be obtained by iteratively solving the aggregate path loss function from a circular disc of radius r_0 at an arbitrary distance $p > d_0$ from the source for τ_l (and τ_u for OLA-T). For an arbitrary choice of γ, the aggregate path loss function is given by: $f(r_0, p) = \int_0^{r_0} \int_0^{2\pi} [(p - r\cos\theta)^2 + r\sin\theta^2]^{-\gamma/2}$, for which there are no closed-form solutions when $\gamma > 2$, and so it is computed numerically.

In order to evaluate FLE under the path loss model assumption for higher values of γ ($\gamma > 2$), we proceed as follows. First, the minimum node degree, $\mathcal{K}_{\text{O,min}}$, for infinite broadcast using Basic OLA is obtained for a disc-shaped network under the continuum assumption. Using Monte-Carlo simulations, we verify these results for random network realizations with finite node densities. Next, the minimum node degree, $\mathcal{K}_{\text{OT,min}}$, which guarantees infinite network broadcast when using OLA-T is obtained for higher path loss exponents. We consider $\gamma = 3$ and 4. For each γ, the OLA boundaries are computed by solving the above equation numerically for Basic OLA and OLA-T, both operating in their minimum power configurations. Using these results, the FLE achieved by A-OLA-T relative to Basic OLA for each γ is obtained. The results along with the details of the simulations are presented in the following sections.

IV. Results and Discussion

A. Simulation Details

The numerical analysis was performed using Matlab. For the continuum case, 1000 radii definitions (levels) were computed iteratively for different values of γ to test for infinite broadcast. We considered $\gamma = 2, 3,$ and 4, and a range of values for the node degree, $\mathcal{K}$. The source power, P_s was chosen to be 3 and the decoding threshold, τ_l was 1. The minimum node degrees for Basic OLA and OLA-T, $\mathcal{K}_{\text{O,min}}$ and $\mathcal{K}_{\text{OT,min}}$, respectively, corresponded to the values of $\mathcal{K}$ at which the radii stopped increasing, i.e., only a finite portion of the network was reached. Additionally, for OLA-T, each $\mathcal{K}_{\text{OT,min}}$ corresponded to a lower bound on RTT, $\mathcal{R}_{\text{lower bound}}$. The Monte-Carlo simulations were used to obtain the results, and assumed 2000 nodes to be randomly and uniformly distributed on a two-dimensional disc of radius 20 distance units with the source node located at the center. A successful broadcast was when 99% of the nodes in the network could decode the message. The Monte-Carlo results were obtained from a simulation of 400 random network realizations. Normalized values were used in each case. The source and relay powers were chosen to be 3 and 0.5, respectively. The decoding threshold, τ_l, and the reference distance, d_0 were assumed to be unity. Nodes in the first level used an $\mathcal{R} = 5.44$ in dB, for all the trials. Lastly, the minimum node degrees to ensure infinite network broadcast for the two node density cases are within 10% of each other, thereby validating the continuum assumption and adding confidence to the numerically obtained results.

B. Minimum Node Degree for Basic OLA, $\mathcal{K}_{O,min}$

Figure 1 is a plot of the probability of successful broadcast (PSB) as a function of node degree for different path loss exponents, γ, for Basic OLA. The following values were consider for γ : $2, 3,$ and 4. The plot shows the simulation to obtain the minimum node degree, $\mathcal{K}_{\text{O,min}}$, for a non-coherent OLA-based cooperative broadcast. $\mathcal{K}_{\text{O,min}}$ is also evaluated for different network density cases, namely the continuum ($\rho \to \infty$) and the finite density. The results for the continuum case are discussed first. The abscissa is the node degree and the ordinate is the probability of a successful broadcast. The step function that represents the continuum assumption is plotted for each γ. It can be observed that as the path loss exponent, γ, increases from 2 to 4, $\mathcal{K}_{\text{O,min}}$ increases from 1.44 to ≈ 3 (black curves without symbols). It is noted that the $\mathcal{K}_{\text{O,min}}$ for $\gamma = 2$ obtained numerically is consistent with [13]. In order to validate the numerical results for the continuum case, we considered random

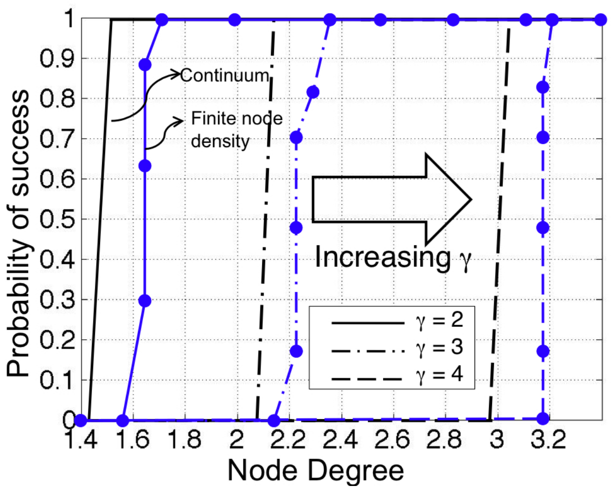

Fig. 1. Probability of successful broadcast (PSB) for Basic OLA for different path loss exponents, γ. The blue and black curves represent the finite node density and continuum cases, respectively.

networks with finite node densities to obtain the $\mathcal{K}_{\text{O,min}}$ for different γ. As expected, the minimum node degree required for a successful broadcast is slightly higher for the finite node density case, and when γ increases from 2 to 4, $\mathcal{K}_{\text{O,min}}$ increases from ≈ 1.6 to ≈ 3.2 for the finite node density case (blue curves). It is noted that the $\mathcal{K}_{\text{O,min}}$ for $\gamma = 2$ obtained numerically is very close to the theoretical value from [13]. Lastly, the minimum node degrees to ensure infinite network broadcast for the two node density cases are within 10% of each other, thereby validating the continuum assumption and adding confidence to the numerically obtained results.

C. Numerical Lower Bounds on RTT for OLA-T

Figure 2 shows $\mathcal{R}_{\text{lower bound}}$, in dB, versus the node degree for different path loss exponents, $\gamma = 2, 3,$ and 4, for OLA-T. These results are for the continuum case only. The results for the $\gamma = 2$ case is the baseline. It can be observed that for a given node degree, the $\mathcal{R}_{\text{lower bound}}$ increases as γ increases from 2 to 4. For example, for $\mathcal{K} = 10$, the minimum transmission threshold is ≈ 0.1 dB for path loss exponent 2 (solid line). However, the minimum transmission threshold is ≈ 1 dB (dash-dotted line) and ≈ 2.2 dB (dashed line) for $\gamma = 3$ and 4, respectively. So the value of $\mathcal{R}$ for sustained OLA propagations when $\gamma = 2$ is insufficient when $\gamma > 2$. Alternatively, this implies that higher node degrees are required for operating OLA-T in its minimum power configuration as γ increases. For example, compared to $\gamma = 2$, there is a 20% increase in the required node degree for infinite network broadcast when $\gamma = 4$. It is also remarked that operating at $\mathcal{R}_{\text{lower bound}}$ may not be very effective if the

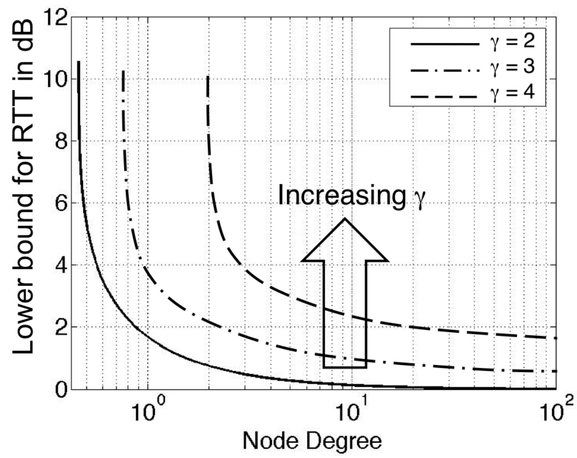

Fig. 2. Lower bound on RTT, $\mathcal{R}_{\text{lower bound}}$, in dB, versus node degree, $\mathcal{K}$, for different path loss exponents, γ, for OLA-T.

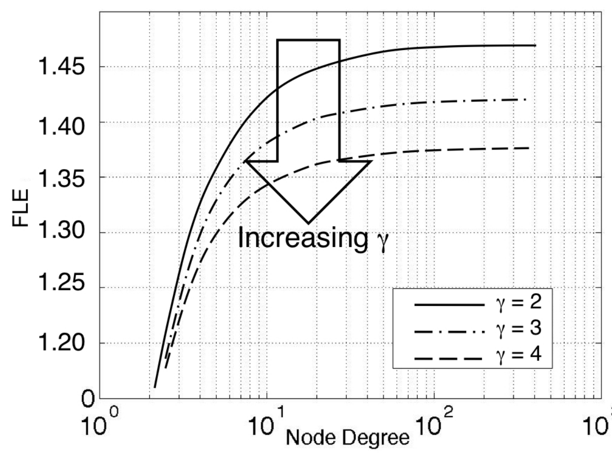

Fig. 3. Variation of FLE with the minimum node degree for a disc-shaped network with 1000 levels for different path loss exponents.

precision in the estimate of the SNR is not good enough. All these factors increase the thickness of the OLAs in each hop/energy consumption of OLA-T at higher path loss exponents, thereby affecting the FLE of A-OLA-T. We remark that same numerical lower bounds were obtained for the infinite disc and strip networks for different path loss exponents. This is not surprising since similar conditions for sustained broadcast held for path loss exponent, $\gamma = 2$, for both the disc and strip networks using Basic OLA [5], [13] and OLA-T [6]. Lastly, it is remarked that the optimum A-OLA-T performance is achieved when $\mathcal{R}_{\text{lower bound}} = \mathcal{R}_{\text{upper bound}}$ at $\mathcal{K}_{(\text{OT,min})}$ [8].

D. Fraction of Lifetime Extension

Figure 3 shows FLE versus minimum node degree, $\mathcal{K}_{(\text{OT,min})}$ (on a logarithmic scale), for a disc-shaped

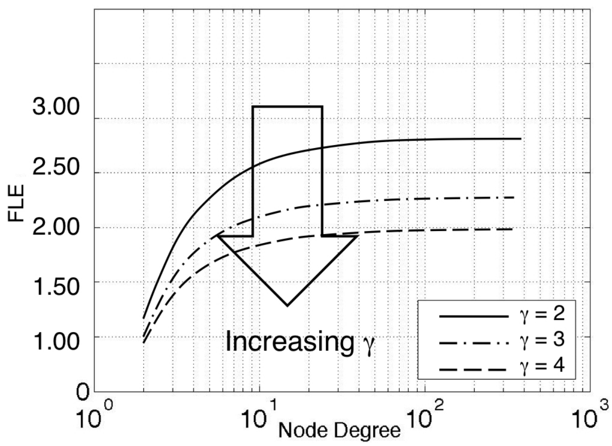

Fig. 4. Variation of FLE with the minimum node degree for a strip-shaped network with 1000 levels for different path loss exponents.

network with 1000 levels for different values of γ. For example, when $\gamma = 2$ (solid line), at $\mathcal{K}_{(\text{OT,min})} = 10$, FLE is about 1.42. This means that at their respective lowest energy levels at $\mathcal{K}_{(\text{OT,min})} = 10$, A-OLA-T extends network life by about 142% relative to Basic OLA. On the other hand, for $\mathcal{K}_{(\text{OT,min})} = 10$ and $\gamma = 4$ (dashed line), the FLE is about 1.34, meaning that A-OLA-T offers a network life extension of about 134% relative to Basic OLA, both protocols operating in their minimum power configurations. It is noted that FLE increases with $\mathcal{K}_{(\text{OT,min})}$ and attains a maximum of about 142% (for $\gamma = 3$) and about 137% (for $\gamma = 4$).

Figure 4 shows FLE versus minimum node degree, $\mathcal{K}_{(\text{OT,min})}$ (on a logarithmic scale), for a strip-shaped network with 1000 levels for different values of γ. For example, when $\gamma = 2$ (solid line), at $\mathcal{K}_{(\text{OT,min})} = 10$, FLE is about 2.50. This means that at their respective lowest energy levels at $\mathcal{K}_{(\text{OT,min})} = 10$, A-OLA-T extends network life by 250% relative to Basic OLA, more than that obtained for a disc-shaped network, which should not be surprising. On the other hand, for $\mathcal{K}_{(\text{OT,min})} = 10$ and $\gamma = 4$ (dashed line), the FLE is about 1.80, meaning that A-OLA-T extends network life by 180% relative to Basic OLA, both protocols operating in their minimum power configurations. It is noted that FLE increases with $\mathcal{K}_{(\text{OT,min})}$ and attains a maximum of about 238% (for $\gamma = 3$) and about 200% (for $\gamma = 4$).

V. Conclusions

A-OLA-T is still a smart and energy-efficient alternative compared to Basic OLA for repeated network broadcasts in a static network. However, it was observed that increasing the path loss exponent, γ from 2 to 4 required more hops to achieve network broadcast with slightly thicker OLAs implying increased node participation in each hop. Increasing γ increased the minimum relay transmit power per unit area, $\overline{P_r}_{(\text{OT,min})}$ (and subsequently, the minimum node degree, $\mathcal{K}_{(\text{OT,min})}$) and the lower bound on RTT, $\mathcal{R}_{\text{lower bound}}$ for sustained network broadcast using OLA-T. Therefore, the fraction of transmit energy saved by OLA-T relative to Basic OLA decreased at high path loss exponents. This implies that for very lossy channels ($\gamma > 4$), the energy savings would diminish considerably, and the performance of A-OLA-T would approach that of Basic OLA during repeated network broadcasts.

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